TESTING COMMAND MODIFICATIONS TO GRAPH A VECTOR FIELD OVER A CONE IN 3D USING MAPLE

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Abstract

Undoubtedly science researchers have benefited from the arrival of mathematical software by different manufacturers such as MATLAB (™), Maple (™), Mathematica (™) and the like. The fast reactivity of these software tools to changes in their parameters allows researchers to observe and study mathematical models in real time and speed up their research process. It is desirable that these wonderful tools be improved and applied to a wide range of new applications. However, one important aspect to consider is the difficulty of translating the traditional mathematical formulas as written in books or manuals into the appropriate syntax required by the mathematical software. In the present paper, the authors show the difficulties to graph a vector field over a cone in 3D using both, an outdated version of the mathematical software Maple (™) and its latest version. Although a solution was obtained the authors think that this solution is far from being optimal. This work includes details of the code modifications and the combination of instructions or commands to obtain the desired solution.

Keywords: maple software, plot a cone, vector field over a cone, 3d vector field

JEL Classification: C02

1. Introduction

One powerful resource in the research world is to test a combination of scientific processes such as that of visualizing a vector field over a surface. It is widely known that visualizing a combination of mathematical functions may help explain many physical events.

Tools that permit us to make multiple quick tests over on complex simulations of world nature are very appreciated in the scientific field.

Recently the authors, using the mathematical software Maple(™) [1], have been studying moving particles on a membrane hoping to find a mathematical model that shows the

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relationship between the membranes' surface areas and the quantity and direction of the particles pushed by them in a gaseous medium. Usually, we find examples in textbooks about how a vector field passes through a surface but not what happens when the field itself starts from a surface.

One of the many inconveniences we have experienced when working with mathematical software is the waste of time reconciling the syntax of the mathematical expressions as written in books or the Internet with that of the required software. An example using Maple is the instruction shown in Figure 1 and its corresponding cone of Figure 2. Notice that the traditional cone equation is not reflected in the code of the figure.
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Maple code to plot a Cone

```
plots:-display(plot3d([rho, theta, (1/4)*Pi], rho = 0 .. 3*sec((1/4)*Pi), theta= -Pi .. Pi, coords = spherical));
```

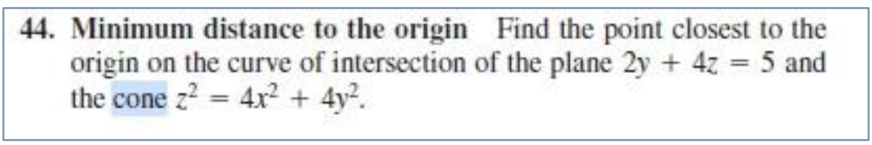
44. Minimum distance to the origin Find the point closest to the origin on the curve of intersection of the plane $2y + 4z = 5$ and the cone $z^2 = 4x^2 + 4y^2$.

Figure 1 Cone equation as presented in a book. [2]

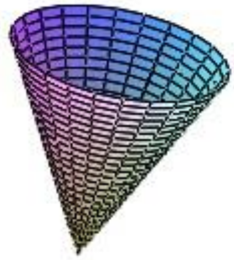

Figure 2 3D cone plotted on Maple

Let us consider another Maple software snippet using a different instruction to plot a cone anew. As seen, the classical cone equation is not reflected in the instruction syntax.

```
restart; x := r*cos(phi); y := r*sin(phi); plot3d([[x, y, r], [x, y, 4]], r = 0
.. 4, phi = 0 .. 2*Pi, style = surfacewireframe, color = grey, scaling =
constrained, labels = ["x", "y", z])
```

When using Maple, we experienced several difficulties when plotting a vector field over a surface. Fortunately, on the Internet we found a help file which after several trials and modifications allowed us to plot a vector field through a cone. This procedure is explained next.

1. PLOTTING A VECTOR FIELD ON A SURFACE WITH MAPLE.

The following code snippet is the one we found on the Internet. We also show the plot it produces. See figure 3.

```
with(plots): with(VectorCalculus):
SetCoordinates(cartesian[x, y, z]):
eqn := (x^2+3y^2)*exp(1/2*(-x^2-y^2)):
montagne := plot3d(eqn, x = -3..3, y = -3.6.. 3.6, shading = zgrayscale, grid = [300,300],
scaling = unconstrained):
normals := Gradient(z-eqn):
display(montagne,seq(seq(arrow([x, y, eqn],normals/2),colour = red),
x = -3..3),y =-3..3), scaling = constrained, axes = boxed)
```

```
> with(plots) : with(VectorCalculus) :
  SetCoordinates(cartesian[x, y, z]) :
  equal := (x^2 + 3*y^2)*exp(1/2*(-x^2-y^2)) :
  model := plot3d(equal, x = -3 .. 3, y = -3.6 .. 3.6, shading = xyz,
    grid = [300, 300], scaling = unconstrained) :
  normals := Gradient(z - equal) :
  display(model, seq(seq(arrow([x, y, equal], normals/2, colour = red),
    x=-3 ..3), y=-3 ..3), scaling = constrained);
```

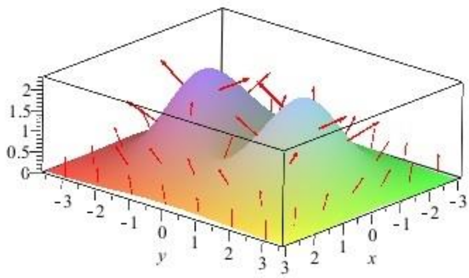

Figure 3 Vector field through a surface

When we tried to modify the equation of the previous snippet using the Cartesian expression of a cone to write it as shown in our reference book the software did not allow us to do so.

The authors also tried to modify the plot of vector field in 3D as produced by the Vector Calculus Tutorial in the Tool menu of Maple. This was not possible either. The next window, Figure 4, shows the only plot that we were able to obtain.

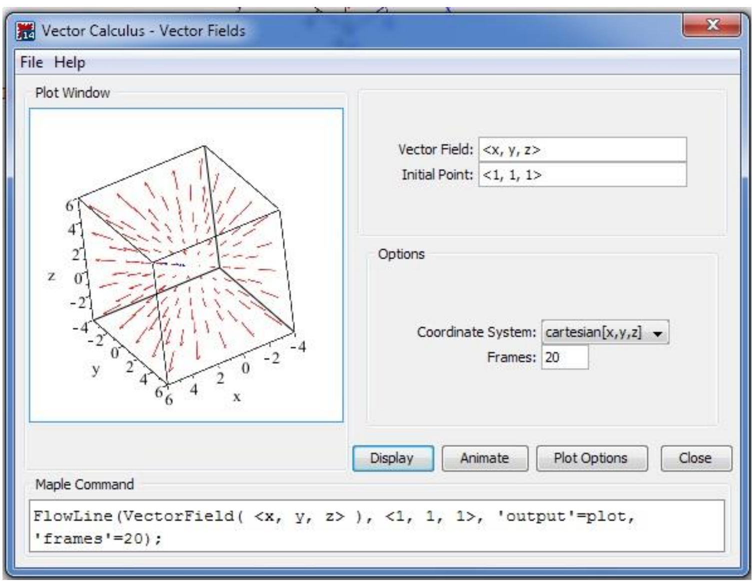

Figure 4 Vector Field on Maple using the Vector Calculus Tutorial

2 MODIFYING THE CODE SNIPPET

The following step illustrates the modifications implemented in the snippet of Figure 3.

Changing The Coordinates System and the Equation

The first try we did was to replace the equation of Section 1.

$$(x^2 + 3y^2) * e^{\left(\frac{1}{2}(-x^2 - y^2)\right)}$$

By the Cartesian expression of the cone equation

$$x^2 + y^2 = z^2$$

After several trials we did not obtain the desired results. Therefore, we decided to change the Cartesian coordinates to spherical and proceed to write the conversion of the variables x and y

$x = r * \cos(phi) \quad y = r * \sin(phi)$

As a result, we obtained the cone's graph, but the vector field disappeared (See Figure 5).

```
with(plots): with(VectorCalculus):
SSetCoordinates('spherical'r, Φ, Θ):
x := r*cos(phi): y := r*sin(phi)):
model := plot3d([x, y, r],r = 0..4,phi = 0..2*Pi, shading = xyz, grid = [300,300], scaling =
unconstrained, axes = boxed):
normals := Gradient(z-model):
display(model, seq(seq(arrow([x, y,r*sin(phi)+r*cos(phi)],normals/4), colour = red), x =-
2..2,y =-2..2,scaling =constrained);
```

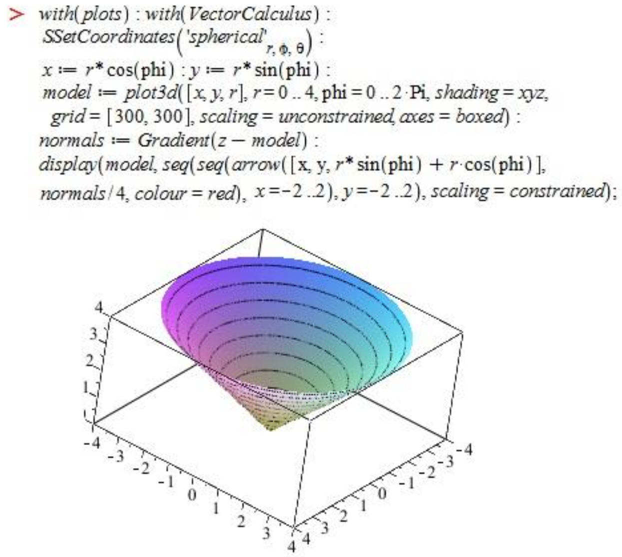

Figure 5 First try to plot a vector field on a cone

Changing The Limit of the Equation

The membrane we wanted to study is that of an electrodynamic loudspeaker with a cone shape. To do this we decided to change the limits of the equation from r = 0..4 to r = 1..4 of the cone to get a frustum (See Figure 6). We also set the third variable of the display function to 5 in lieu of the spherical expression (changes are shown in blue). The result we obtained, at this scale, is the representation of the vector field as a set of points.

```
with(plots): with(VectorCalculus):

SSetCoordinates('spherical'r, Φ, Θ):
x := r*cos(phi): y := r*sin(phi):
model := plot3d([x, y, r], r =1.. 4, phi = 0..2*Pi,shading = xyz,
grid = [300, 300],scaling = unconstrained, axes = boxed):
normals := Gradient(z-(model):
display(model,seq(seq(arrow([x, y, 5], normals/4), colour =
red), x = -2..2, y = -2..2),scaling = constrained)
```

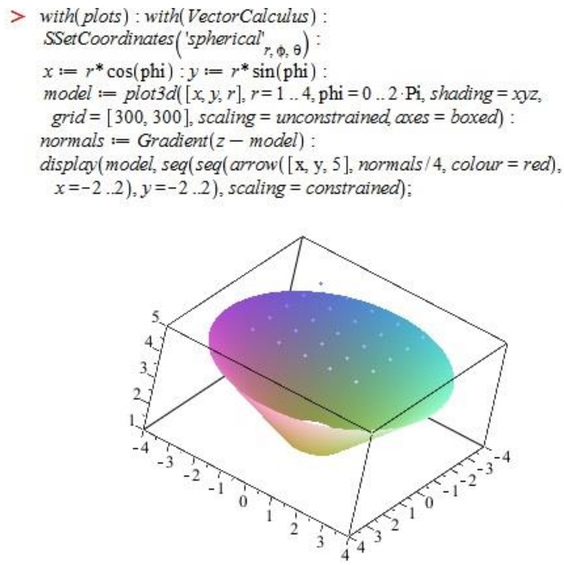

Figure 6 First view of the vector field over the cone

2.3 Obtaining a Visible Vector Field

The visualization of the vector field was obtained by increasing the variables x and y. This allowed us to obtain a wider view of the plot. Finally, we adjusted the plot by subtracting 2 from the axes x and y respectively. We also increased the third variable of the display function to 7 as shown in Figure 7. Likewise, all changes are shown in blue. However, the whirlpool vector field obtained did not show the direction that we expected.

```
with(plots): with(VectorCalculus):
SSetCoordinates('spherical'r, Φ, Θ):
x := r*cos(phi): y := r*sin(phi):
model := plot3d([x, y, r],r =1..4,phi = 0 ..2*Pi,shading = xyz,
grid = [300, 300],scaling = unconstrained, axes = boxed):
model1 :=fieldplot3d([4*x-2, 4*y-2,6], r = -4..4, phi =
-Pi..Pi,z =4..7):
normals := Gradient(z-model):
display(model,model1, seq(seq(arrow([x, y, 7], normals/4), colour=
red), x = -2..2), y = -2..2,scaling = constrained;)
```

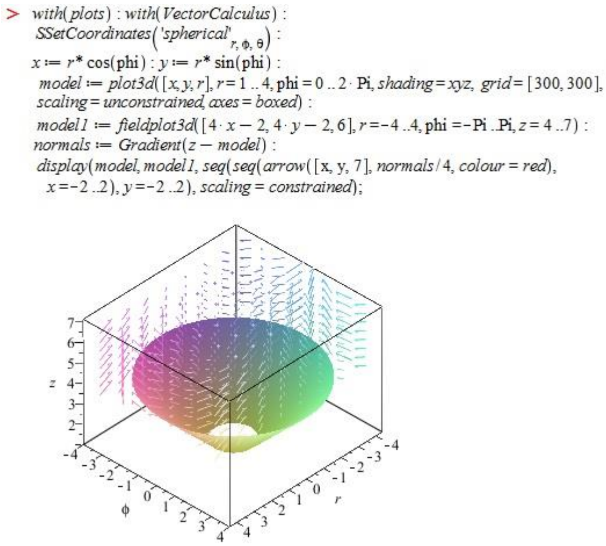

Figure 7 Whirlpool vector field over a cone

3 Looking for the Appropriate Vector Field

In our next step, we proceeded to look for a vector field with a unique origin point [4]. Fortunately, in the help file of the software was an example with the desired field's shape (See Figure 8).

```
> fieldplot3d([1, 0, 0], r = 0 .. 4, t = 0 .. (1/2)*Pi, p = 0 .. (1/2)*Pi, coords = spherical, axes = boxed);
```

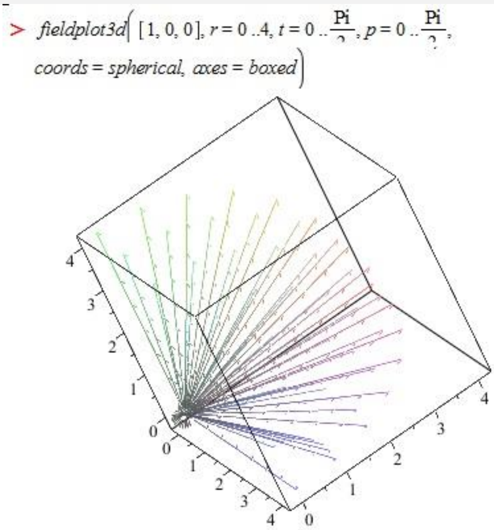

Figure 8 Vector field from the origin

3.1 First View of The Directed Vector Field Over the Cone

When we tried to integrate this command with the previous one, we did not know if these two were going to be compatible. Some adjustments were necessary to achieve the correct view. In this respect, in model 1, we changed the axes of Fieldplot3D by switching t and r to make it coincide with the direction of the cone (See Figure 9). Finally, we got the directed vector field over our cone, but still it did not have the correct width that we expected.

```
with(plots): with(VectorCalculus):
SSetCoordinates('spherical'r, Φ, Θ):
x := r*cos(phi):y := r*sin(phi))
model := plot3d([1.9*x,1.9*y,r-4], r = 1 .. 4*phi = 0 ..2* Pi, shading = xyz, grid = [300, 300], scaling = unconstrained, axes = boxed):
model1 := fieldplot3d([1, 0, 0], r = 0 .. Pi/1.5, t = 2 .. 7, p = 0 .. Pi/1.5), coords = spherical, axes = boxed, color = blue):
normals := Gradient(z-model):
display(model, model1, scaling = unconstrained);
```

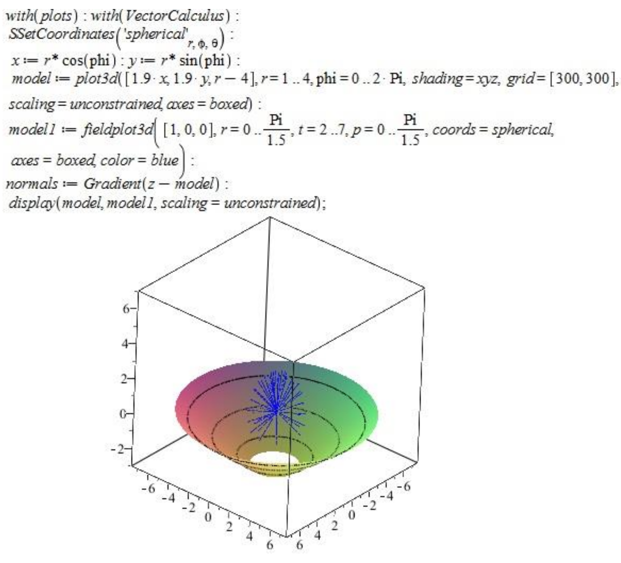

Figure 9 Directed Vector Field over The Cone

3.2 Final View of the Desired Vector Field Over the Cone

Based upon our partial success by obtaining a vector field coming from a cone loudspeaker membrane, we decided to modify the equations of the model and model1 and by tweaking the appropriate parameters we finally got the desired graphic as shown in Figure 10.

```
with(plots): with(VectorCalculus):
SSetCoordinates('spherical'r, Φ, Θ):
x := r*cos(phi)): y := r*sin(phi)):
model := plot3d([0.4*x,0.4*y,r/8],r =0.8..3, phi=0..2*Pi, shading = zgrayscale,grid = [300, 300], scaling = unconstrained, axes =boxed): model1:= fieldplot3d([1,0,0.5], r =0..Pi/2.8, t = 4..9,p = 0..Pi/2.8, coords = spherical,axes = boxed, color = blue):
normals := Gradient(z-model):
display(model, model1, scaling = unconstrained);
```

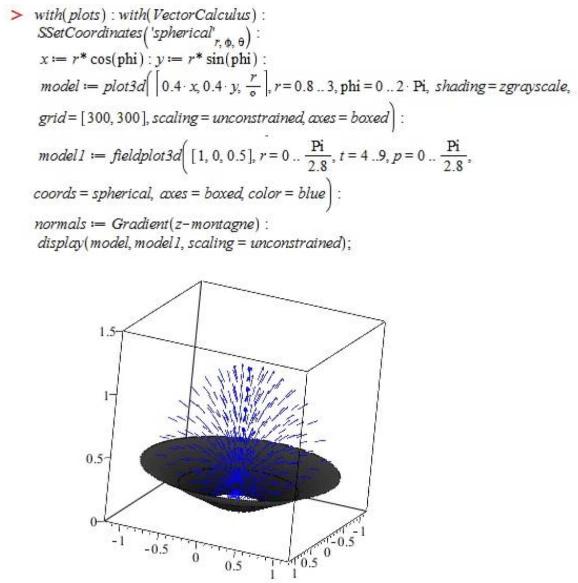

Figure 10 Our desired vector field over the cone

4. CONCLUSIONS

It is important to mention that the aim of this work is to encourage researchers and practitioners to employ powerful tools that improve their research without spending too much time learning how to use these tools. The learning curve to master the language and functionality of mathematical software may vary from weeks to months and, in some cases, years. For researchers mastering a tool is not the main objective of their endeavors. Tools are just an auxiliary complement to facilitate research and share the findings with the interested community.

It is common to encounter new auxiliary technologies and tools that can help us to teach our experiences, however, some of them may be easy to learn while others may have a long learning curve. Finding shortcuts that can speed up the learning process are always helpful and welcome.

This work's field of application extends beyond studying a loudspeaker membrane. For example, inverting the direction of the vector field could be used to study the tympani membrane or microwave parabolic antennas.

It is important to note that the Maple commands and codes written in this paper do not necessary match the exact syntax of Maple. What this means is that if you copy any piece of code and paste it on a worksheet, it may not work. This is because we exported them from Maple as plain text and then edited them manually.

References

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